Transport demand elasticities estimated by discrete choice models

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1 Introduction

The key purpose of surveying travellers' choice of travel mode is to quantify behaviourally meaningful relationships between the mode-choice and a set of explanatory variables. These relationships are useful for measuring sensitivity to policy which affects the level of modal attributes at the disaggregate level. However, when evaluating policy, decision-makers are concerned with aggregate demands, because they affect public transport revenue, the environment and traffic congestion. The analyst is aware of the policy requirements for studies into the choices of individuals and reports aggregate summaries. Most importantly s/he will report a set of aggregated elasticities as a demand forecast tool for evaluation of policy.

> *"A major innovation in the analysis of transport demand was the development of disaggregate travel demand models based on discrete choice analysis methods. … We are clearly concerned, however, with the forecasting of aggregate demands." (Ben-Akiva and Lerman 1985 p1).*

The market effect of switching behaviour between modes is represented by an elasticity matrix for a conditional demand system. The theoretical derivation of these demand systems is random utility maximisation (RUM) which differs from the usual economists' theory of consumer behaviour primarily because of the type of data under analysis; an individual always chooses a single good (corner solution), and not portions of many goods. In each choice setting, a single alternative - including delaying travel and no travel - is chosen from a set of mutually exclusive and exhaustive possibilities (Louviere et. al. 2000, Train 2003).

As an extension to the choice model, investigating consumer choices over time is the discrete\continuous demand model (Hanemann 1984). In such models, the consumer is assumed to simultaneously select one good from a group of substitutes and the quantity of that good to consume. Examples include brand choice (Chiang 1991), appliance choice and energy demand (Dubin and McFadden 1984) and subscription to and use of a service (Madden et al 1993). In the latter two examples, the choice component is an initial investment in a technology. Similar methods have been used to study automobile demand (Hensher et. al 1992 and Train 1986). Each of these studies model the ownership and change-over of vehicles within a household as well as the kilometre usage of the vehicles. Where as, Hensher et. al. maintained a four year panel, Train's study used a cross-sectional sample with a question about their usage and ownership of vehicles for the previous twelve months.

While it is desirable to have inter-temporal observations, either as repeat choices or usage over time, some studies take a cross-section sample at a single point in time. These studies are usually performed with a single purpose in mind. For example, to estimate a matrix of direct and cross-price elasticities (Hensher and Raimnond 1995). The discussions and theories herein relate to such data and had their beginnings when working with the commuter segment of the Hensher data (Taplin, Hensher and Smith 1999). The main finding being the properties of choice models that hold at the individual level, do not necessarily hold at the aggregate level.

The extension here is to infer conditional ordinary demand functions from probability choice systems. The method uses an expected expenditure for the group before and after a change in the price of one alternative.

The restrictions on aggregate behaviour as modelled by discrete choice models are quite different from the restrictions on aggregate behaviour imposed in traditional price-income demand system estimation. Where price-income models estimate substitution elasticities conditional on an income constraint, discrete choice models estimate substitution elasticities subject to a fixed aggregate quantity constraint. The difference between the two approaches to demand modelling is presented as a decomposition of the price elasticities. The pure discrete choice model estimate demand elasticities conditional on a constant marginal utility of money. The remaining price response is due to a recalculation of the consumer's budget allocated to transport.

The paper is organized as follows: after a discussion on discretionary travel demand separability is offered as an important tool allowing the analyst to study the demands of interest (i.e. transport) without needing detailed information about off-sector prices. Two stage budgeting is presented as an explanation of the consumer's decision process consistent with separability. Next discrete choice models and their behavioural outputs are discussed. The elasticity estimated by discrete choice are presented as demand elasticities, where the consumer is given sufficient compensation as to keep his marginal utility of money constant. In the last two sections the demand for transport is considered as a sector of expenditure in the consumer's budget and the demand elasticities are given as the sum of within group (mode) substitution and between group (transport) substitutions.

2 Discretionary travel demand models

A behavioural model used for policy analysis contains essential detail about the problem for action to be taken. We do not expect the model to be explicit about every aspect of the behaviour it is meant to represent. When "no new intuitions emerge about the problem" a model for decision support is requisite (Phillips 1984). Requisite models have sufficient detail to allow confidence in the results, but are not so intricate as to obscure the interpretation of the results. For example, when setting fare levels public transport authorities rely on estimates of traveller's response to examine policy outcomes such as revenues or traffic congestion (i.e. decrease or increase of car use). The decision maker is interested in an aggregate demand response that is an accurate reflection of the decisions made by the individuals affected by the policy.

Discrete choice models (DCM) are the class of statistical models used to infer relationships between a discrete response variable and a vector of explanatory variables. The models can be used on wide class of preference data, including preference ordering, scaled ratings, and choosing one or some from many (Louviere et al 2000 pp 25-33). A *discrete choice* is where an individual selects one alternative given a finite set of alternatives. In a utility maximising framework the chosen alternative is the one which offers the highest level of satisfaction to the individual. The *pure discrete choice model* provides a description of aggregate demand when the total number of trips (choices) is fixed and the analysis can proceed as if the aggregate demand is the sum of individual choices. This would describe a population of mostly full time car users and mostly full time public transport users, such as the commuting market. In the short term, at least, the nature and quantity of each trip is fixed. The effect of rising public transport fares is felt primarily at the extensive margin where some individuals are switching from public to private transport.

For discretionary travel neither the quantity nor the characteristic of the trips undertaken by the individual are set. The non-commuter will consider the type and location of activities to undertake as well as the frequency of participation in these activities. Each trip is interrelated, where the individual budgets both time and money allocated to activities linked together by trips (Ben-Akiva and Bowman 2001). While the aggregate demand for transport is the collection of the individual's decisions, the richness of the consumer's choice set means that the full effect of a price change in any of the modal alternatives is not adequately explained by individuals switching between modes. The move from transport supply management to travel demand management has seen an increasing need for models of travel behaviour to capture a more complete picture of the household's decision process (Kitamura and Fujii 1998). Here policy measures as diverse as flexible working weeks, telecommuting, parking restrictions or the inclusion dedicated bus lanes rely on a very detailed understanding of travel behaviour. Pricing is just one of many policy instruments. However, it may be possible to analyse consumer responses to price changes without the need of such detailed models, by adequately capturing the consumer's decision process to a level of detail sufficient to estimate the change in market demand in transport. The essential behaviour for analysis of the non-commuter market is apparent at two levels. The consumer will evaluate the available mode alternatives taking into account their total cost for a given level of demand and at the same time the consumer will decide on the level of demand given the prices of the chosen mode alternative. In a sense the consumer makes a decision of what to buy and how much to buy. This decision balances the utility derived from travel with the money left over for other forms of expenditure.

In a mode choice setting the alternatives are perfect substitutes. Each may offer different levels of service, but their underlying function serves the same purpose. The price the consumer is willing to pay for a particular level of service is reflected in rate of exchange between money (i.e. consumption of other goods) and the attributes offered by one or another mode. For the current market conditions, the consumer decides on this rate of exchange. This decision takes into account the value of the attributes as well as the value of other forms of expenditure. Distinct from other modal attributes, travel cost and travel time play unique roles in the consumer's decision process. They reflect the amount of other activities, goods and services the consumer is willing to forgo to afford a particular mode. Their units of measure are freely exchangeable between one consumption activity and another, yet the individual has a limit to the total amount of each resource and therefore she budgets between consumption activities. The focus of this paper is on the response to travel costs.

If the price for travel on one mode increases the consumer will make an adjustment to the allocation of money to travel and other expenditure. It is convenient to decompose this adjustment into two components. First, they revaluate the comparative position of the modal alternatives in their utility function. Assume they do this using the observed rate of exchange between money and non-price attributes. In effect their value of money has not changed; later this is referred to as a constant marginal utility of money. At this point we may consider the total number of trips as fixed and the response in the market is the switching between modes. This represents the same market response as the commuter market. Second, for those affected by the price change there is a higher price to bear for travel. In this case these consumers' will need to recalculate how much money they will allocate to travel at the new price. While the consumer is free to make this readjustment in anyway they seem fit, two important cases are worth mentioning.

1. The consumer may choose to continue to make the same trips as before the change. To pay for the additional travel costs the consumer will need to reduce expenditure on other consumption by the amount equal to the additional travel cost. This behaviour is described by a demand function where the elasticity for travel is zero and the cross-price elasticity for non-travel goods with respect to price of travel is minus one. If the consumer were to behave in this way, then the discrete choice model will provide an adequate representation of market demand. The only relevant behaviour is the switching between modes due to the price change. Importantly, the fixed demand assumption also implies that the income elasticity for travel is also zero.

2. The consumer may choose to continue to allocate the same amount of money to the consumption of travel as before the change. In effect they have constant demand for all other goods. This describes the situation where the elasticity of demand for travel is minus one and the cross-price elasticity with respect to travel price is zero. Think of this as the situation where the consumer has a fixed demand for non-travel consumption items.

While neither situation may be an accurate representation of the consumer's actual budgetary response, each play an important role in models developed to capture a depiction of a consumer's decision when detailed information is required about a small section of the consumer's expenditure. The assumptions embodied in these restrictions on behaviour are non-limiting, because at any observation the consumer may reveal which variant of the discrete alternative is preferred as well as the level of demand. In doing so the consumer reveals the budget allotment for travel and non-travel expenditure. However, the analysis of policy requires the model to estimate the changes in demand given changes in market conditions (e.g. price). It is important to recognise the market response curves estimated by models of demand for a section of the consumer's expenditure usually fall into situation 1. (McFadden 1981) or situation 2. (Pollak, 1971). The demand functions are known as "conditional demand functions" (Pollak 1969) and are applicable where the consumer's utility function is separable in transport and non-transport consumption.

3 Separability and two stage budgeting

When examining a component of consumer behaviour, such as travel, the inclusion of prices of seemingly unrelated consumption goods, such as a particular item of clothing or a pound of rice, will pose such an unnecessary difficulty that the analysis may never proceed. As such travel behaviour analysts have examined the travel choices in relative isolation from other consumption activities. By doing so, they have incorporated *separability* in their studies. "Separability is about the structure we are to impose on our model: what to investigate in detail, what can be sketched in with broad strokes without violation to the facts", Gorman (1987).

Separability is a partitioning of the utility function groups. The marginal rate of substitution between goods within a group is independent of the quantities demanded for goods not belonging to that group. For example, my substitution of bus patronage for train rider-ship is independent of the number of books upon my shelf. However, that is not to say that the consumption of books does not limit the amount of money available for the complier of travel. The idea was conceived independently by Leontief (1947) and Sono (1962) and it was Strotz (1957) who first published the application of separability to a two stage budgeting process. This is where the consumer sees the budget allocation in two-stages. Firstly, expenditure is allocated to broad groups, such as food, housing and transport. Secondly, the sector's budget is allocated to products within the sector, such as meat, eggs and fish. The budgeting process need not be restricted to two levels, as shown in figure 1.

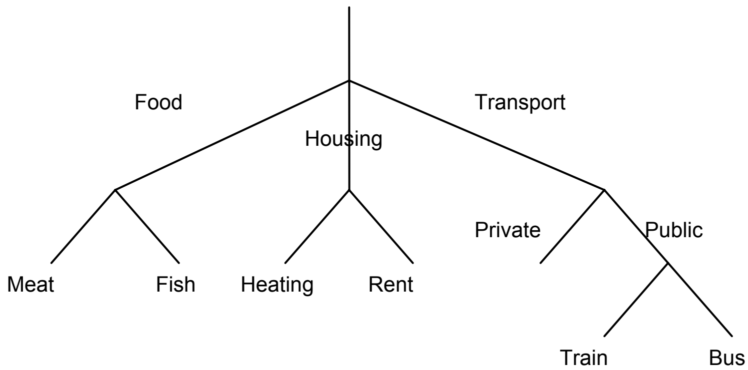

Figure 1: The multi-stage budgeting process represented by a tree diagram.

The main implication for demand studies of the "utility tree" is that there is no specific substitution between goods within alternate branches of the tree. The main functionality of *meat* can not be replaced by a greater consumption of *train* or visa versa. The only interaction between such goods is through their competition for the consumer's budget. The way in which the consumer evaluates the relative benefits of private and public transport is independent of whether more meat or more fish is consumed, all that matters is the amount of expenditure the consumer has for consumption of transport.

Separability is both necessary and sufficient for lowest stages (bottom of the tree) of a multi-stage budgeting process (Gorman 1970). The way in which the consumer organises his expenditure on transport is a function of expenditure allocated to transport and transport prices. The demand functions may be concentrated on the group of commodities under study (e.g. transport alternatives). The demand functions for transport are conditional on the level of consumption of non-transport goods and therefore the budget available for the transport decision. The term conditional demand function seems to be first used Pollak (1969), where one good, which may be a composite of many goods[1], is said to be "pre-allocated" and therefore the income allocated to the transport sector is conditional on the quantity of the pre-allocated good.

To establish the relationship between ordinary demand functions and conditional demand functions, suppose the quantity of the pre-allocated good is precisely equal to the quantity demanded had the consumer been free to choose this amount – essentially the conditional demand specification has placed no additional limits to the consumer's choice. An observation of a consumer's demand is consistent with his total utility function. However, the demand curves are estimated as if the consumer has a fixed demand for non-transport. The demand elasticities for transport are subject to a budget constraint given for the current allocation of money to the transport sector. Should the price of one or more of the transport alternatives rise, the model provides estimates of a re-adjustment of transport expenditures provided that the total expenditure on transport does not change. The conditional demand approach describes situation 2 given in the previous section. The conditional demand functions provide information on how the consumer may apportion his transport budget, but not the total response to a price change. This will need to include information about how the consumer assigns money to the broad groups of expenditure (i.e. the first stage of the two stage budgeting process). The price and income effects for two-stage budgeting are established (Gorman 1970) and an accessible summary is provided in Deaton and Muellbauer (1980). The use of conditional demand functions to establish the same results are given in Pollak (1971) with a summary provided in Pollak and Wales (1992). The main results are presented in section 5.

The practical benefit of separable utility functions is that it reduces the quantity of information needed to study demand. In particular, it allows estimation of utility consistent demand as functions of within group prices and the budget allocated to the group. Two stage budgeting provides a qualitatively reasonable theory of the way a consumer may go about purchasing decisions which is consistent with a separable utility function.

In the next section the discrete choice model in a demand setting is presented. For the same reasons of practicality the consumer's utility function is assumed to be partitioned between transport and other forms of expenditure. The utility specification used for discrete choice analysis is actually more restrictive than the results given in this section. The utility function is additively separable with a constant marginal utility for expenditure on the non-transport commodities. This represents a utility function that is linear for one area of the consumer's budget (non-transport), but nonlinear for another (transport). The utility function of this type is

[1] In figure 1 the composite commodity is the off transport sectors housing and food.

known as the *quasilinear* utility function (Varian 1992) and this is the topic of section 5. In the following section a brief outline of the pure discrete choice model is given.

4 Discrete choice models in a demand setting

The discrete choice model (DCM) may be applied to a wide class of problems; wherever the choice set is a finite set of mutually exclusive alternatives (Train 2003). Under certain conditions the DCM will generate a probability choice systems consistent with random utility maximisation (see Ben-Akiva and Lerman 1985 or Anderson et al 1992 for general discussions). The random component of the consumer's utility may be attributed to the lack of information about the choice available to the analyst (Manski 1977) or it may be that the consumer does behave randomly by focusing on different aspects of the choice on different occasions (Quandt 1956).

Demand for marketable goods, such as travel, is a special case of choice where the consumer will account for the residual income available for purchase of other goods. The class of discrete choice models applicable to consumption activities are known as *conditional DCM's* (McFadden 1981 and Hanemann 1984). These models include the discrete/continuous demand model and the *pure discrete choice* in which the individual consumes a fixed quantity of the discrete alternative.

The general parametric form of DCMs is:

$$\pi_{\mathrm{j}} \equiv \pi(\mathrm{j}|\mathrm{J},\mathrm{s}) = f_{\mathrm{j}}\left(\mathbf{p}^{\mathrm{T}},\ \mathbf{p}^{\mathrm{-T}},\ \mathbf{a}^{\mathbf{T}},\ \mathbf{s},\ \mathrm{m}\right) \qquad (1)$$

Where π_j is the probability that alternative j is chosen from a set of J alternatives, **p** is a vector of all prices segmented into transport prices (T) and non-transport prices (-T), **a** is a vector of non-price attributes for the alternatives (these are transport related) and **s** is the vector of the individual's characteristics of which income (y) indicates the limit of monetary resource available for all expenditure. If the probability choice system (1) is consistent with random utility maximisation, then π_j is also the probability that alternative j is the alternative with the highest ranking in the consumer's utility function.

Let Q^{nT} represent the total quantity of transport demanded by the n^{th} individual, the pure discrete choice model applies a quantity constraint such that Q^{nT} is volume of transport activity both before and after a change in price. The quantity constraint is explicit in pure discrete choice models and is most commonly incorporated by specifying a population of size N where each individual demands exactly one item, such that $Q^T = N$, where Q^T is the sum of all individual choices. The following presents a representative consumer approach to the pure discrete choice model (McFadden 1981 and Anderson et. al. 1992). The solution to consumer's maximising problem is given by the following indirect utility function.

$$\overline{V}\left(p^T, a^T, z, s, m, \varepsilon\right) = \max_{x^T, z} U\left(x^T, a^T, z\right) + \varepsilon \;;\; z \le m - p^T \cdot x^T,\; x^T = Q^T \qquad (2)$$

Where the indirect utility function $\overline{V}$ is a function of the price (p^T) and qualitative attributes (a^T) of the transport alternatives, the cost of purchasing z units of the composite commodity, the social-economic variables (s) of which m is reserved for the individual's monetary budget. The consumer chooses the discrete alternative such that a sufficient level of income is available to purchase z units of the off-sector consumption goods. The random component of utility (ε) represents either the unobserved component of the individual's utility or the distribution of tastes/decision rules throughout the population.

To assist with the discussion, consider one individual with fractional consumption rates such that $Q^{nT}=1$. Think of this as an individual consumer who chooses one variant of the transport modes. To expand back to the market level we multiply by N; where N maybe thought of as the total number of trips within the market. On choosing the preferred mode the consumer has revealed his preference and has $y-p_i$ income remaining for expenditure on non-transport consumption, where p_i is the price of the trip using mode i (the superscripts n and T are dropped). Assume the consumer's conditional indirect utility function is additive:

$$V_i = \lambda\left(m - p_i\right) + \beta \mathbf{a}_i + \varepsilon_i \qquad (3)$$

Where λ is a constant representing the marginal utility of money and the β's are *weights* reflecting the comparative importance of each attribute. The utility expression contains a random variable ε_i and therefore is itself a random variable. Should the consumer be maximising his/her utility then the generated probability choice system indicates the probability that mode i is the mode with the maximum utility.

$$\pi_i \equiv Pr\left(V_i > V_j \;\; \forall j \in J\right) \qquad (4)$$

The *additive* separability structure of U allows the model to represent the consumer's mode choice as a function of mode attributes and mode prices. The information about detailed prices for all non-transport consumption is sketched in as the consumer's demand for money. The consumer accounts for his entire budgetary decision as in (2), even though the choice model includes only information about the modes. This acts in the same way the conditional demand functions discussed in section 3, where the functions are given as group prices and group expenditure, but are meant to represent to consumer's complete demand decision. The observed choice is consistent with the consumer's total utility for consumption. The discrete choice model provides estimates of this choice up to a probability.

The model provides two important economic outputs. The ratio β/λ indicates the amount of money available for non-transport expenditure the consumer is willing to forgo for an improvement on one of the non-price attributes of the modal alternatives. This is easily equated by setting the total differential dv_i to zero, representing a change in the mix of attributes without making the consumer better or worse off. The ratio is known as the willingness-to-pay (Hensher et. al. 2005). An important example of the willingness-to-pay measure is the value of travel time savings, in which β is the marginal utility of time. It should be noted, that the ratio is only appropriate for infinitesimal changes in the non-price attribute.

To calculate a willingness-to-pay measure for a large change in any attribute (including price) welfare calculations are appropriate. The advantage of the linear in income utility function is the change in the indirect utility function (2) is easily equated as the loss (gain) in income available for non-transport expenditure. However, because the model estimates choice (the alternative with the highest utility) up to a probability, this calculation is usually performed using an expected value (McFadden 1981). The calculation of this measure assumes that the consumer's demand for transport stays the same after the change in attributes. Assume that mode i is selected and think of the expression $+\lambda p_i - \beta \mathbf{a}_i$ as the cost of purchasing one unit, if the price is increased by one unit, the loss in utility can be compensated by an additional amount of income valued at λ per dollar[2]. As the consumer is unable to adjust the demand for transport, the loss in utility is evaluated at the loss in money available for non-transport expenditure. As an aside, the loss (gain) in utility is greater for that of the constrained demand given by a pure discrete choice model than had the consumer been free to re-adjust his/her demand (see Deaton and Muellbeaur 1980 section 4.3 for a discussion on demand with constraints on purchasing). These measures represent two of the three behavioural outputs of a choice model; the third is the elasticity of choice.

The elasticity of choice represents the proportional change in probability that mode i is the mode with the highest utility with respect to a percentage change in one of the attributes for that that mode or any other of the modes. The elasticity of choice on mode i with respect to the price of mode j is given as:

$$\frac{\partial \ln \pi_i}{\partial \ln p_j} \equiv \frac{\partial \ln Pr\left(V_i > V_j \quad \forall j \in J\right)}{\partial \ln p_j} \tag{5}$$

The elasticity of choice is not necessarily the elasticity of demand except under certain conditions.

Consider each consumer having a fixed level of trips, Q^{nt}. For simplicity we will consider that each individual makes all trips by the mode offering the highest utility[3]. Should the price of any alternative change, for some people their current mode may no longer be the one yielding the highest utility. These people will convert all their Q^{nt} trips to an alternate mode. The choice elasticity is indicative of the demand elasticity as the quantity of demand has been diverted to a competing mode. The elasticity of market demand is an appropriate aggregation of the disaggregate elasticities (see Ben-Akiva and Lerman 1985). Determining market demand elasticities from individual choice elasticities is appropriate for the commuter market, where each individual has a fixed level of demand, Q^{nt}= 10 trips per week, say. The market response is the sum of the individual responses which may be estimated from the individual choice elasticities.

For the discretionary travel market the total effect of a price change is a composite of mode switching, for which choice elasticities can reveal estimates, and changes in the quantity of demand, for which further information is required. The discrete choice model may be thought of as a conditional demand under separability, i.e. a solution to the second stage of a two stage budgeting process. However, in contrast to the conditional demand functions discussed in section 3, where the non-transport demand is fixed, the discussion to follow considers the problem where the demand for transport is fixed. The discussion relies on the properties of a *quasilinear* utility function which are given in the following section.

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[2] Because the probability choice system only identifies the mode with the highest utility up to a probability, the relevant price is the expected value over all modes, i.e. $-\mathrm{Exp}\left[\max_{i}\left(-\lambda p_i + \beta \mathbf{a}_i\right)\right]$

[3] This need not be the case. A person or household may have a portfolio of mode choices for different trip characteristics. The limitations of the data collected for mode choice studies means the unit of analysis is the trip. The market demand is the sum of individual trips without accounting for the interdependencies between trips.

5 The Quasilinear utility function and conditional demand functions

Consider a consumer with fractional consumption rates for modes 1, 2, …J who demands Q^{nT} units of transport. The demand for mode i is δ_{in}, where, $\delta_{in} = \pi_{in} \times Q^{nT}$, is the outcome of the probability choice system and may be thought of as the expected demand determined by the consumer's solution to the random utility problem[4]. The consumer is assumed to evoke a two stage budgeting process for determining demands for transport and all other expenditure, and therefore has a utility function separable in these items. Further it is assumed that utility component has sufficient structure for which a price index of transport, p^T, is defined; this is a technical point first made by Gorman (1959).

The deterministic indirect utility function corresponding to (2) is given as:

$$\overline{V}\left(p^T, z, s, m\right) = V^*\left(p^T; s\right) + m \;;\; z \le m - p^T \cdot Q^{nT} \qquad (6)$$

Where p^T may be thought of as a generalised price or a utility weighted price which incorporates the non-price attributes. However in the discussion to follow it is defined as an expected market price which accounts for the attributes by way of the consumer's choices (i.e. the probabilities).

The indirect utility (6) is a function additively separable into a term independent of p^T and a term independent of total income. The stochastic counterpart can be found in McFadden (1981) equation 5.13. Essentially, it implies that the demand for transport is a function of transport prices and is independent of the level of income available for consumption on other goods. "The approach is fine, but it hides a potential problem. Upon reflection, it is clear that demand for [transport] can't be independent of income for all prices and all income levels. If income is small enough the demand for [transport] must be constrained by income". (Varian 1992 p 164).

The demand functions for the quasi-linear utility are of the form

$$x^T\left(p^T\right) = w^T \frac{m}{p^T} \qquad (7)$$

Where, w^T is the proportion of income allocated to transport.

If z =0, then w^T =1 and the corresponding direct utility is $U(m/p^T)$. Think of starting the consumer at m=0 and increasing income by a small amount. The change in utility is $\frac{\partial U/\partial m}{p^T}$.

If this is greater than 1, then the consumer is better off spending the first dollar on transport rather than the composite commodity. The consumer will continue to spend additional income on transport until the marginal utility of consumption is equal to the price of transport. Any additional income will be spent on z.

In figure 2, the theoretical Engel curve (in bold) turns a right angle at the income level for which the marginal utility is equal to price. The second curve (ON`X) is arbitrary, however, it is meant to be a more realistic representation of a consumer's income curve. In either case the curve is vertical at the current income level for which X is demanded. The demand for transport is given as OR and the demand for z is OP. If the any additional income is available to the consumer, it will all be spent on z. On the other hand if the budget constraint is MN, the consumer's transport demand is ON (theoretic) or ON'(approximate). Any additional income increases the demand for transport. It is clear that the indirect utility function (2 or 6)

[4] The *expected* market demand, Q^T, for mode i is the sum over all δ_{in} for the population

is a local approximation to the consumer's preferences only valid at income levels for which the income elasticity for transport is *approximately* zero.

The price effects for this model are worth mentioning. Say the consumer has income m = XQ, and the price for transport is m/Q. If the price for transport increases to m/Q', the consumer will reduce expenditure on z by R/M(Q-Q') so that the demand for transport remains at R after the price change. Similarly, if the price for transport falls, the consumer will allocate the cost savings to z. The price elasticity for transport is also *approximately* zero.

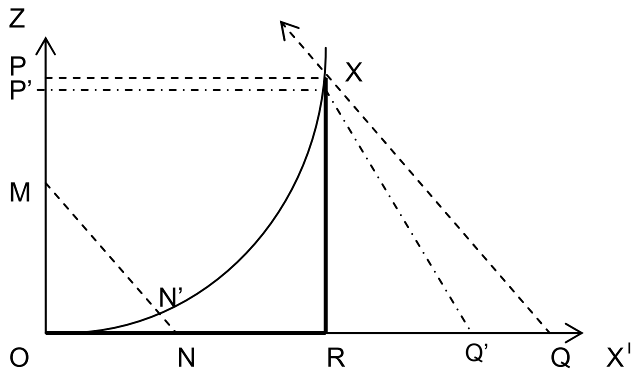

Figure 2: Engel (income) curves for a quasi-linear utility function

The quasilinear utility function effectively describes situation 1. from section 2. The consumer continues to make the same trips as before the change. To pay for the additional travel costs the consumer will need to reduce expenditure on other consumption by the amount equal to the additional travel cost.

The model of behaviour described by the quasilinear utility function is "often used in applied welfare economics since …demand only depends on price – at least for large enough incomes – as there are no income effects to worry about" (Varian 1992 p 165). This is most surely the advantage McFadden (1981) made use of when developing the social surplus description of aggregate (market) preferences. Perhaps the easiest way to interpret the use of a quasilinear utility function for calculating welfare measures is to look at the problem from a direct utility viewpoint. Let's say the consumer derives benefit from making trips and the consumption of all else, the direct utility corresponding to (6) is

$$U\left(Q^T, z, s, m\right) = U^T\left(Q^T; s\right) + \lambda z \; ; \; z \le m - p^T \cdot Q^T \qquad (8)$$

If the price for transport increases the consumer will continue to demand Q^T items in which case there is no change in U^T, any loss in utility is through the budget constraint leading to a decreased amount of z being purchased. For example if the price for transport is increased by one dollar, the loss to the consumer may be repaid by exactly one dollar in compensation, because all of this additional dollar will be spent on z. Therefore the welfare calculations are equal to the change transport price multiplied by current demand.

However, when applied to discretionary travel demand, the model places an unwarranted restriction on the consumer behaviour in that the level of travel demand may not be adjusted to account for price adjustments. Even though the consumer may reveal which variant of the discrete alternative is preferred as well as the level of demand, the elasticities estimates are conditional on the utility derived from transport is constant, because the same quantity appears in U^T, as is the marginal utility for money (i.e. the marginal expenditure on z). In the demand literature this is known as the Frisch elasticity (Frisch 1959) which is seldom used in applied literature (except see Clements 1987).

A possible way forward is to draw an analogy to applied work in consumer demand described in section 3 of this article. Recalling the main difference between the two approaches is that in traditional demand estimation under two stage budgeting the conditional demand functions are defined by a quantity constraint for the composite commodities and in the quasilinear approach the constraint applies to the collective of goods under study, namely transport.

In the next section the two approaches to demand studies for a group of competing goods are contrasted by way of the properties of their elasticities. The derivations of these results are quite lengthy and refer the reader to Smith (2006) for the theoretical discussion.

6 Elasticities

The mode-choice elasticities estimated by discrete choice models may be thought of as conditional demand elasticities where the consumer is given sufficient compensation to remain at the current level of demand. In the setting above, this corresponds to compensation such that the consumer's marginal utility of income is constant. The total (ordinary) demand elasticity is the composite of the switching between modes as well as the substitution between transport and all other expenditure. While the argument leading to this point is somewhat different, the result does not differ from earlier comment on this topic (Quandt 1968 and Taplin 1982). The relationship between an ordinary elasticity ε_{ij} and the corresponding choice elasticity m_{ij} is:

$$\varepsilon_{ij} = m_{ij} + \frac{\partial \log Q^T}{\partial \log p_j^T} = m_{ij} + \eta_j \qquad (9)$$

Where, Q^T is the aggregate demand for transport and P^T_j is price on mode j. The second term on the right, η_j, is the generation or second stage elasticity (Taplin 1982). Taplin sets out a method for calculating mode-choice elasticities from ordinary elasticities; however this results in a loss of information and would rarely be a useful exercise (Taplin 1982). To infer ordinary elasticities from mode share elasticities information is needed about the elasticity for the aggregate demand for transport and the elasticity of the price index P^T with respect to the price on, say, the j^{th} mode (Oum et. al. 1992).

$$\eta_j = \frac{\partial \log Q^T}{\partial \log P^T}\frac{\partial \log P^T}{\partial \log p_j^T} \qquad (10)$$

Starting with the elasticity of the transport price index with respect to the price on mode j, this may be thought of as the rate in which additional money will need to be allocated to the transport sector to keep the demand at current levels given a price rise on one of the modes. This result is derived by Smith (2006):

$$\frac{\partial \log P^T}{\partial \log P_j^T} = \omega_j = \sum_{k=1}^{J} w_k \left(m_{kj}\right) + w_j \qquad (11)$$

The elasticity for the transport sector with respect to the price index P^T applies to the upper level of the utility tree, or the first stage of the two stage budgeting process. Assuming all other expenditure can be thought of a single good (Hicksian composite commodity) the own-price elasticity for transport is given as

$$\varepsilon_{TT} = \frac{\partial \log Q^T}{\partial \log P^T} = \phi\varepsilon_{Tm} - \phi w^T \varepsilon_{Tm}\varepsilon_{Tm} - w^T \varepsilon_{Tm} \qquad (12)$$

Where w^T is the proportion of income allocated to transport, and $\phi = \left(\frac{\partial \lambda}{\partial m}\frac{m}{\lambda}\right)^{-1}$ is

known as the Frish price flexibility, which is the inverse of the elasticity of utility with

respect to income.

There is a strong case for ϕ=-0.5. Selvenatan (1993) reports and estimate of ϕ=-0.46 for Australia and a pooled result of ϕ=-0.45 for 15 OECD countries. Theil (1987) obtains a value of ϕ=-0.53. More importantly, Theil's model rejects Frisch's original conjecture that ϕ decreases (becomes more negative) systematically as income increases. This is supported by Clements and Theil (1996), and Selvenathan (1993). However, Dejanvry et al (1972) and Lluch et al (1997) report findings in support of Frisch's conjecture.

7 Conclusion

Mode-choice elasticities differ from ordinary demand estimates because they do not take into account the change in the total volume of transport (Oum et. al. 1992). Through a decomposition of ordinary demand elasticities Taplin (1982) identified the difference between the two elasticity measures was a demand generation response; the elasticity for total volume of traffic with respect to the price on mode j. This paper identifies the micro-economic theory behind the demand generation elasticity. If the mode choice elasticities are thought of conditional demand estimates using a quasi-linear separability utility, the generation elasticities are the results obtained through the first stage of the two-stage budgeting process.

The main result is given by

$$\varepsilon_{ij} = m_{ij} + \left(\sum_{k=1}^{J} w_k m_{kj} + w_j\right)\left[\phi\varepsilon_{Tm} - w^T \varepsilon_{Tm}\left(1 + \phi\varepsilon_{Tm}\right)\right] \qquad (15)$$

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